MATH 53
Quiz 7, October 24th, 2007 Name: _____

Answer no more than two of the following questions, indicating clearly which two you would like graded by circling their numbers.

1. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$

   Solution: The region we are integrating over is the region under the curve $y = x^2$ as x ranges from 0 to 1. So switching the order of integration gives $\int_0^1 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx = \int_0^1 y\sqrt{x^3+1}\,|_0^{x^2}\,dx = \int_0^1 x^2\sqrt{x^3+1}\,dx = \int_1^2 \frac{1}{3}\sqrt{u}\,dx = \frac{2}{9}u^{3/2}\,|_1^2 = \frac{2}{9}(2\sqrt{2}-1)$.

2. Use polar coordinates to evaluate $\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3+xy^2)\,dy\,dx$.

   Solution: To find the area of integration, plot the curves $y = -\sqrt{9-x^2}, y = \sqrt{9-x^2}$, which give a circle of radius 3 around the origin. However, we only let x vary from 0 to 3, so we will only get the right half of the circle. To convert to polar coordinates we first note that $x^3+xy^2 = x(x^2+y^2) = r\cos\theta \cdot r^2$. In polar coordinates we then have $\int_{-\pi/2}^{\pi/2} \int_0^3 r^3\cos\theta \cdot r\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \int_0^3 r^4\cos\theta\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^5}{5}\cos\theta|_0^3\,d\theta = \frac{3^5}{5}\cdot\sin\theta|_{-pi/2}^{\pi/2} = 2\cdot\frac{3^5}{5}$.

3. Consider the lamina that occupies the region bounded by the parabola $x = 1-y^2$ and the coordinate axes in the first quadrant. The density at a point (x,y) is proportional to its distance from the x-axis. Find the y-coordinate of the center mass of the lamina, given that the mass of the lamina is $k/4$.

   Solution: The distance from a point to the x-axis is the y value of the point. Hence $\rho(x,y) = ky$. Then by using the formula for the y-coordinate of the center of mass, we have $\frac{4}{k}\int_0^1 \int_0^{1-y^2} y\cdot ky\,dx\,dy = 4\int_0^1 \int_0^{1-y^2} y^2\,dx\,dy = 4\int_0^1 y^2\cdot x|_0^{1-y^2}\,dy = 4\int_0^1 y^2 - y^4\,dy = 4(y^3/3 - y^4/4)_0^1 = 4(1/3-1/5)$.