MATH 53
Quiz 5, October 3rd, 2007 Name: ________________________

Answer no more than two of the following questions, indicating clearly which two you would like graded by circling their numbers.

1. Evaluate the following limits, if they exist, or show that the given limit does not exist. Justify your answers.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{2x^3+y^3}, \qquad \lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$$

   Solution: For the first one, the limit along the x-axis is zero, while the limit along the line $y = x$ is $1/3$, so the limit cannot exist.

   As for the second, note that this problem cannot be solved by checking the limit through every line through the origin. The easiest to way to show the limit exists and is zero is to see that if $(x, y) \neq (0, 0)$, then $\frac{x^4-y^4}{x^2+y^2} = \frac{(x^2-y^2)(x^2+y^2)}{x^2+y^2} = x^2 - y^2$. Then since $x^2 - y^2$ is continuous at $(0, 0)$, the limit is 0. This problem can also be done by showing that $\frac{x^4}{x^2+y^2}$ and $\frac{y^4}{x^2+y^2}$ both have limits of zero at the origin, and then subtracting them. One could also use polar coordinates.

2. Find the tangent plane to $z = e^x \cos y$ at the point (a, b). Use that formula to find the tangent plane at $(0, 0)$

   Solution: The tangent plane formula is $z = f(a, b)+f_x(a, b)(x-a)+f_y(y-b)$. We have $f_x = e^x \cos y$, $f_y = -e^x \sin y$. Thus $f_x(a, b) = e^a \cos b$, $f_y(a, b) = -e^a \sin b$, and $f(a, b) = e^a \cos b$. The tangent plane is $z = e^a \cos b+e^a \cos b(x-a)-e^a \sin b$. If $(a, b) = (0, 0)$, we get $z = 1 + x$.

3. Determine if $f(x, y) = \frac{x}{x+y}$ satisfies the partial differential equation $f_x - f_y = \frac{1}{x+y}$.

   Solution: Using the quotient rule, $f_x = \frac{y}{(x+y)^2}$ and $f_y = \frac{-x}{(x+y)^2}$. Then $f_x - f_y = \frac{x+y}{(x+y)^2} = \frac{1}{x+y}$.