MATH 53

Quiz 4, September 26th, 2007 Name: ______________________

Answer no more than two of the following questions, indicating clearly which two you would like graded by circling their numbers.

1. What quadric surface is $z = xy$? How can you tell? [HINT: Look at traces $y = x + c$.]

   Solution: Traces $y = x + c$ and $y = -x + c$ give parabolas while traces $z = k$ give hyperbolas (for $k \neq 0$). Thus, the surface is a parabolic hyperboloid, rotated $\pi/4$ radians. We can also perform a change of variables, $x = u + v$ and $y = u - v$ (which is equivalent to a rotation of our axes by $\pi/4$ radians) to obtain $z = u^2 - v^2$, a hyperbolic paraboloid.

2. On a single graph, sketch three level curves of $f(x, y) = y \sec x$. [HINT: Should level curves intersect?]

   Solution: The level curve for a particular value k is the set of points which satisfy $k = y \sec x$. In particular, we cannot have $\cos x = 0$, since f is undefined there. So the level curves are of the form $y = k \cos x$ except we must exclude the points where $\cos x = 0$. The excluded points are exactly those where the level curves would intersect if not restricted. In general, level curves can never intersect, since this would imply that a point (x, y) takes on multiple values, which cannot happen since f is a function.

3. Find where the curves $\mathbf{r}_1(t) = \langle \cos t, \sin t, 0 \rangle$ and $\mathbf{r}_2(s) = \langle 1, s^2 - 1, \ln s \rangle$ intersect. Calculate $\cos\theta$, where θ is the angle the curves make at a point of intersection.

   Solution: First we must find the intersection of the two curves. That is we must find t_0 and s_0 such that $\mathbf{r_1}(t_0) = \mathbf{r_2}(s_0)$. For this to happen we must have $\ln(s_0) = 0$ hence $s_0 = 1$. We also need $\cos t_0 = 1$, so t_0 must be a multiple of 2π. Since $\mathbf{r_1}$ is periodic, we may take $t_0 = 0$. We have been forced to choose $t_0 = 0$ and $s_0 = 1$ so that the first and third coordinates of $\mathbf{r_1}$ and $\mathbf{r_2}$ match. Luckily, the second coordinates match also when $t_0 = 0$ and $s_0 = 1$. Now the angle of intersection of two curves is the angle between their tangent vectors. So we compute $\mathbf{r_1}'(t_0) = \langle -\sin t_0, \cos t_0, 0 \rangle = \langle 0, 1, 0 \rangle$ and $\mathbf{r_2}'(s_0) = \langle 0, 2s_0, 1/s_0 \rangle = \langle 0, 2, 1 \rangle$. Then $\cos\theta = 2/\sqrt{5}$ by using the dot product formula.