MATH 53
Quiz 2, September 12th, 2007 Name: ________________________

1. (3 points) Find a polar function $r = f(\theta)$ whose graph is the line $x + y = 1$.

   Solution: We use the identities $x = r\cos\theta, y = r\sin\theta$. After substituting, we have $r\cos\theta + r\sin\theta = 1$. Solving for r yeilds $r = \frac{1}{\sin\theta+\cos\theta}$.

2. Find the area of the intersection of the regions formed by $r = 3$ and $r = 6\cos\theta$.

   Solution: After graphing the two curves we see the problem is finding the intersection of two circles of radius three, one with center $(0,0)$ and one with center $(3,0)$. One can do this with basic trigonometry, but we will take polar curve approaches here. The intersection of the two curves occurs when $6\cos\theta = 3 \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = -\pi/3, \pi/3$. Thus we'll calculate the area under the $r = 3$ curve from $-\pi/3$ to $\pi/3$ and add to it the area under the second curve. However, we must note that the area we wish to add is traced out by the graph of $6\cos\theta$ as θ moves from $\pi/3$ to $2\pi/3$. So we calculate the area as $1/2\int_{-\pi/3}^{\pi/3} 3^2 d\theta + 1/2\int_{\pi/3}^{2\pi/3}(6\cos\theta)^2 d\theta = 1/2(9\theta)\ |_{-\pi/3}^{\pi/3} + 1/2\int_{\pi/3}^{2\pi/3} 18\cos 2\theta + 18 d\theta = 3\pi + 1/2(9\sin 2\theta + 18\theta)\ |_{\pi/3}^{2\pi/3} = 3\pi + 1/2(9\sqrt{3} + 18\pi/3) = 6\pi - 9\sqrt{3}/2$.

   Alternatively, we can note that the region is symmetric about the line $x = 3/2$, so we may calculate the area by finding the area between the curves $r = 3$ and $x = 3/2$, which we rewrite in polar coordinates as $r = \frac{3}{2}\sec\theta, -\pi/2 < \theta < \pi/2$. The intersection of these curves occurs when $\cos\theta = 1/2 \Rightarrow \theta = -\pi/3, \pi/3$. So we calculate $2 \cdot \frac{1}{2}(\int_{-\pi/3}^{\pi/3} 3^2 - (\frac{3}{2}\sec\theta)^2 d\theta = 9\theta - \frac{9}{4}\tan\theta\ |_{-\pi/3}^{\pi/3} = 6\pi - \frac{9}{2}\sqrt{3}$