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Practice Final, Math 1B

MULTIPLE CHOICE

1. Which of the following is correct for any convergent series $\sum_{n=1}^{\infty} a_n$ with positive terms?
   (a) $\left(\sum_{n=1}^{\infty} a_n\right)^2 = \sum_{n=1}^{\infty} a_n^2$
   (b) $\sum (a_n)^{1/n}$ converges.
   (c) $\sum x^n a_n$ has a radius of convergence R satisfying $R > \epsilon$ for small, positive epsilon.
   (d) $\sum (a_n + \epsilon)^2$ converges for small enough ϵ.

2. The recurring decimal $0.234234234234\ldots$ is the rational number
   (a) 234/999
   (b) 234/111
   (c) 117/999
   (d) 234/499

3. The radius of convergence of the power series $f(x) = (x-1)^5$ is
   (a) 1
   (b) ∞
   (c) 0
   (d) 1/5

4. Which of the following will find a particular solution to the differential equation $y'' + y = 1/e^x + e^x$ most easily?
   (a) Try a solution of the form $y = Ae^x - Ae^{-x}$.
   (b) Try a solution of the form $y = A\cosh x$.
   (c) Try a power series solution, $y = \sum a_n x^n$.
   (d) Try $y = u_1(x)\cos x + u_2(x)\sin x$.

5. Suppose z is a complex number satisfying $z^n = 1$. Which of the following is false?
   (a) $z = e^{i2k\pi/n}$ for some k.
   (b) $z\overline{z} = 1$.
   (c) If we write $z = a + bi$, then $-1 \le a \le 1$ and $0 \le b \le 1$.
   (d) $z + \overline{z}$ is real.

6. The indefinite integral $\int x^2 e^{x^2} dx$ is

   (a) $\frac{e^{x^2}}{2} + C$
   (b) $\frac{xe^{x^2}}{2} + C$
   (c) cannot be expressed in terms of finitely many elementary functions
   (d) $\frac{x^2}{3} e^{x^2} + 2x^3 e^{x^2} + C$.

7. Which of the following is true for any sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = 2$?

   (a) There is an $N > 0$ for which $a_n < 2$ for all $n \geq N$.
   (b) There is an N for which $|a_n - 4| < 20$ for all $n \geq N$.
   (c) For no value of n is a_n larger than $2/\epsilon$.
   (d) For any $\epsilon > 0$, there is an N with $|a_n - 4| > \epsilon$ for all $n \geq N$.

8. Consider the series $\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n}$ which of the following is correct?

   (a) The series converges absolutely.
   (b) The series converges, but not absolutely.
   (c) The series does not converge because the nth term does not tend to 0.
   (d) The series does not converge, by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

9. Let $f(x) = e^{x^2-4x}$. Then $f^{(50)}(2)$ is

   (a) $1/e^4$
   (b) $50!/25!$
   (c) $e^{-4} 50!/25!$
   (d) 0.

10. The integral $\int_{-\infty}^{\infty} (e^{-x^2} - 1)/x^2$ is

   (a) $\sqrt{\pi}$.
   (b) divergent
   (c) convergent
   (d) we can't tell if it's convergent or divergent because the integral cannot be done in finite terms

11. The general solution to the differential equation $y'' + 4y = 0$ is

   (a) $y = c_1 \cos t + c_2 \sin t$.
   (b) $y = \cos 2t + \sin 2t$.
   (c) $y = \sin(2t + \theta)$.
   (d) $y = c_1 e^{2t} + c_2 e^{-2t}$.

12. The equation $2 + i = re^{i\theta}$ holds for

   (a) $r = 3, \theta = \pi/6$.
   (b) $r = \sqrt{3}$, $\theta = \pi/3$
   (c) $r = \sqrt{3}$, $\theta = 13\pi/6$
   (d) $r = \sqrt{3}$, $\theta = \pi/4$.

13. Which of the following is false concerning a linear homogeneous second order differential equation with constant coefficients?

   (a) The boundary value problem $y(x_0) = 0$ and $y(x_1) = 0$ always has a solution.
   (b) The initial value problem always has a unique solution.
   (c) The boundary value problem may not have a solution, but if one exists it is unique.
   (d) The boundary value problem may not have a solution, and if one exists it may not be unique.

14. Let $\{a_n\}$ be any sequence. Then the series $\sum a_n 2^{-np}$

   (a) Converges for large enough p
   (b) Converges for large enough p if $a_n \geq 0$ for every n
   (c) Converges if the a_n alternate and satisfy $a_{n+1} < 2^p a_n$ and have $\lim_{n\to\infty} a_n = L$ for some number L.
   (d) Converges if $\lim_{n\to\infty} a_n = 0$ and the a_n alternate.

15. The length of the curve $y = e^{|x|}$ for $-1 \leq x \leq +1$ is

   (a) $\int_{-1}^{1} \sqrt{1 - e^{2|x|}} dx$
   (b) $2\int_{0}^{1} \sqrt{1 + e^{2x}} dx$
   (c) $2\int_{-1}^{1} \sqrt{1 - e^{2x}} dx$
   (d) $\int_{-1}^{1} \sqrt{1 + e^{|x|}}$

FREE ANSWER

1. Give two linearly independent solutions to $y'' - xy = 0$.

2. A vat has a volume of 1000 liters. It initially contains 250 liters of pure water. Brine with a concentration of 2 grams per liter begins to flow into the vat at a rate of 25 liters per minute. The mixed solution escapes through a leak at a rate of 5 liters per minute. How much salt is there in the vat when it is full?

3. Solve the initial value problem $y'' - 2y + y = e^x$, $y(0) = 0$, $y(0) = 1$.

4. Is the series $\sum \frac{1}{n^2+3n+2}$ convergent? If so, find its sum.

5. Suppose f is a continuous positive strictly decreasing function for $x \geq 1$ and $a_n = f(n)$. By drawing a picture, rank the following in increasing order:

$$\int_1^k f(x)dx, \quad \sum_{i=1}^{k} a_i, \quad \sum_{i=2}^{k} a_i.$$

Taking limits as $k \to \infty$, state and prove the integral comparison test for series.

6. Find a particular solution to the differential equation

$$y'' - 2y' + y = \frac{e^x}{1 + x^2}.$$

7. Evaluate the integral $\int_0^4 x^2\sqrt{16-x^2}$.

8. Evaluate the integral $\int \frac{x}{x^2-x+6}\,dx$.

9. Evaluate the integral $\int (\ln x)^2 dx$

10. Find the sum of the series $\sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n$