MATH 53
Midterm 1 Review Questions, October 5th, 2007

Exam 1

1. Find the area of the region enclosed by the curve $r = 2 + 2\cos(\theta)$.
2. Calculate the tangent of the angle between the vectors $a = (2, 3, -6)$ and b= $(2, 3, 6)$.
3. Let P be the point $(2, 3)$ and Q the point $(7, -4)$. Find the coordinates of the point 3/4 of the way from P to Q.
4. Find the slope of the line tangent to the curve $r = \cos(3\theta)$ at the point $\theta = 3\pi/4$.
5. Show that the curve $x = t^3 - 4t, y = t^2$ intersects itself at the point $(0, 4)$, and find the slopes of the lines tangent to the curve at that point.

Exam 2

1. Sketch the curve given in polar coordinates by
$$r = \left|\ln\frac{\theta}{2\pi}\right|$$
for $0 < \theta \leq 2\pi$.
2. Find the parametric equations of the line perpendicular to the plane $x+2y+3z = 4$, passing through the point $(5, 6, 7)$.
3. If the limit
$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 2y^2}{x^2 + y^2}$$
exists, find it. If it doesn't, say why it doesn't.
4. If x is given implicitly as a function of y and z by the equation $x = y\sin(z+x^2)$, find $\frac{\partial x}{\partial z}$ and $\frac{\partial x}{\partial y}$ in terms of x, y and z.
5. Graph the plane curve
$$x^2 + 2x = 4y^2 - 4y.$$
6. (a) Find an equation of the plane passing through the points $P(1, 2, 0)$, $Q(0, 1, 1)$, and $R(1, 1, 1)$. (b) Compute the area of the triangle PQR.

Exam 3

1. Sketch the parametric curve $x = t, y = 3t^3 + 4t$, for $t \geq 1$. Find the length of this curve from $t = 1$ to $t = 2$.

2. Sketch the polar curve $r = 1 - \cos(2\theta)$. Find the total area enclosed by this curve.

3. Sketch and describe the surface $x^2 + 2x - y^2 + z^2 = 0$. Find its tangent plane at the origin.

4. Find an equation for the line of intersection of the planes $x + 2y + z = 1$ and $x - z = 0$.

5. Find the equation of the plane which goes through the points $(1, 0, 0)$, $(0, 3, 0)$ and $(0, 0, 2)$.

6. Find the limit, if it exists, or show that the limit doesn't exist.

$$\text{(a)} \quad \lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}, \qquad \text{(b)} \quad \lim_{(x,y)\to(1,-1)} \frac{x + y}{x + 2y + 1}$$

7. Let $f(x, y) = x^2y + \ln(xy)$. Find f_{xy} and f_{yx} and the linearization of the function at $(1, 1)$.

8. Let $x^2 + yz^2 - z = 0$, where $x(s, t) = \ln(t - s)$, and $y(s, t) = t$. Find $\frac{\partial z}{\partial t}$.