

MATH 1B

Quiz 9, May 12, 2008

Name: _____

Do all of the following problems.

1. Write the general solution to the differential equation

$$ay'' + by' + cy = 0$$

in the following cases.

- (a) The quadratic $ar^2 + br + c$ has two distinct real roots r_1 and r_2 .

$$y_G = c_1e^{r_1x} + c_2e^{r_2x}$$

- (b) The quadratic $ar^2 + br + c$ has one repeated root r_0 .

$$y_G = c_1e^{r_0x} + c_2xe^{r_0x}$$

- (c) The quadratic $ar^2 + br + c$ has two distinct complex roots $\alpha + i\beta$ and $\alpha - i\beta$.

$$y_G = e^{\alpha x}(c_1 \cos(\beta x) + c_2 \sin(\beta x))$$

2. Determine the solutions to the boundary value problem

$$4y'' - 4y' + y = 0, \quad y(0) = 0, y(1) = 0$$

3. Solve the differential equation with initial conditions.

$$y'' + 2y' = 0, \quad y(0) = 0, y'(0) = \frac{1}{2}.$$

The characteristic equation has roots $-2, 0$, so the homogeneous equation has general solution

$$y_C = c_1 + c_2 e^{-2x}$$

Then if $y(0) = 0$, $c_1 = -c_2$ and $y'(0) = \frac{1}{2}$ gives $c_2 = \frac{-1}{4}$ so

$$y = \frac{1}{4} - \frac{e^{-2x}}{4}$$