MATH 1B

Quiz 5, April 11, 2008 Name: ______________________________

Do both of the following problems.

1. Find the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}(10x)^n}{n!}.$$

We first use the ratio test to find the radius of convergence.

$$\lim_{n\to\infty} \frac{\left|\frac{\sqrt{n+1}(10x)^{n+1}}{(n+1)!}\right|}{\left|\frac{\sqrt{n}(10x)^n}{n!}\right|} = \lim_{n\to\infty} \frac{10|x|\sqrt{n+1}}{(n+1)\sqrt{n}} = 10|x|\sqrt{\frac{\lim_{n\to\infty} \frac{1}{n^2} + \frac{1}{n^3}}{\lim_{n\to\infty} 1 + \frac{1}{n}}} = 0,$$

where we were able to distribute the limit since squareroots are continuous, all the limits exist individually and the denominator does not go to zero. In particular, no matter what x is, the ratio approaches zero, so the interval of convergence of the power series is $(-\infty, \infty)$.

2. Find the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{(x-1)^n \ln(n+1)}{2^n}.$$

We use the ratio test to find the radius of convergence first.

$$\lim_{n\to\infty} \frac{\left|\frac{(x-1)^{n+1}\ln(n+2)}{2^{n+1}}\right|}{\left|\frac{(x-1)^n \ln(n+1)}{2^n}\right|} = |x-1| \lim_{n\to\infty} \frac{\ln n+2}{2\ln n+1} = |x-1| \lim_{n\to\infty} \frac{n+1}{2(n+2)} = \frac{|x-1|}{2},$$

where in the last two steps we used L'Hospital's rule. This tells us that the radius of convergence is 2 around the point $x = 1$. The two series at the endpoints are $\sum(-1)^n \ln(n+1)$ and $\sum \ln(n+1)$. These both diverge by the test for divergence. In sum, the interval of convergence is $(-1, 3)$.