Math 113 Homework 4

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There are five problems, due Thursday, October 3.

1. Consider the quaternion group $Q = \{1, -1, i, -i, j, -j, k, -k\}$ with the binary operation as follows:

	1	-1	i	j	k	-i	-j	-k
1	1	-1	i	j	k	-i	-j	-k
-1	-1	1	-i	-j	-k	i	j	k
i	i	-i	-1	k	-j	1	-k	j
j	j	-j	-k	-1	i	k	1	-i
k	k	-k	j	-i	-1	-j	i	1
-i	-i	i	1	-k	j	-1	k	-j
-j	-j	j	k	1	-i	-k	-1	i
-k	-k	k	-j	i	1	j	-i	-1

   (a) For each element of Q, find its order.
   (b) For each element of Q, find its inverse.

   Remark. See `http://mathworld.wolfram.com/QuaternionGroup.html` for more information about the Quaternion group.

2. Consider the quaternion group Q.

   (a) Find all of the cyclic subgroups of Q [Hint: there are five of them.]
   (b) Is there a non-cyclic subgroup?

3. Consider the group $G = Sym_6$. Let $\sigma = (142)(36)$, and let $\tau = (5362)$, both elements of G.

   If the answer is an element of G, you can express your answer either by saying where it sends each element of $\{1, 2, 3, 4, 5, 6\}$ (which was covered in class on Tuesday Sept 24), or in disjoint cycle notation.

(a) What is $\sigma\tau$?

(b) What is $\tau\sigma$?

(c) What are the orders of τ and σ?

(d) What are the inverses of τ and σ?

4. Consider the group $G = Sym_3$.

   (a) Show that $\{id, (123), (132)\}$ is a subgroup of G.

   (b) What well-known group is $\{id, (123), (132)\}$ isomorphic to?

   (c) The set $\{id, (12)\}$ is a subgroup of G (you don't have to prove this). What are its left cosets?

5. Let $(G, *)$ and $(H, \circ)$ be groups, and suppose that G is generated by the set $\{x_1, \cdots, x_n\}$. Let ϕ and ψ be two group homomorphisms from $(G, *)$ to $(H, \circ)$. Show that if $\phi(x_i) = \psi(x_i)$ for $i = 1, \cdots, n$, then ϕ and ψ are the same homomorphism (i.e., $\phi(g) = \psi(g)$ for all $g \in G$).

   Remark. This proves the important fact that a homomorphism is completely deter-mined by what it does to a generating set. [CAUTION: it does not follow, as in the case of linear algebra, that we can define a homomorphism simply by specifying where to send the generators; one has to be careful about possiblerelations the generators may satisfy]