

Practice Midterm 2

1. Determine whether the following series diverge or converge:

a) $\sum_{n=1}^{\infty} \frac{(-1)^n n + 5 + \log(n)}{\sqrt[3]{n^7 + n^2}},$

b) $\sum_{n=1}^{\infty} \frac{n^{2n}}{(1 + 2n^2)^n},$

c) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}.$

2. a) Find the centroid of the region bounded by the curves

$$y = \sqrt{x}, \quad y = x$$

- b) Now imagine that the plane is submerged in water up to the line $y = 2$ (i.e. with the y -axis pointing up out of the water). Find the hydrostatic force on the region described above. Verify "A Theorem of Vinicius and Dan" – that the force on the surface is equal to the pressure at the centroid times the area of the surface.

3. a) Show that the series $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ is convergent.

b) Deduce that $\lim_{n \rightarrow \infty} \frac{n^n}{(2n)!} = 0.$

4. A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after an hour?

5. Write the rational 0.23171717... as a fraction of two positive integers.

Bonus) Does the series $\sum_{n=1}^{\infty} \tan(e^{-n^2})$ converge or diverge?