

# Integration by Parts and Trigonometric Integrals

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(1) Compute

$$\int \cos^2 x \sin 2x \, dx$$

(2) Compute

$$\int \sin^4 x \cos^4 x \, dx$$

(3) Show

$$\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

(4) A finite Fourier series is given by the sum

$$\begin{aligned} f(x) &= \sum_{n=1}^N a_n \sin nx \\ &= a_1 \sin x + a_2 \sin 2x + \dots + a_N \sin Nx. \end{aligned}$$

Show that the  $m$ th coefficient  $a_m$  is given by the formula

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx \, dx.$$

(5) Use integration by parts to show that for  $n \geq 2$ ,

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \cos^{n-2} x \, dx$$

Show that for odd powers of cosine

$$\int_0^{\pi/2} \cos^{2n+1} x \, dx = \frac{2 \cdot 4 \cdot 6 \dots \cdot 2n}{3 \cdot 5 \cdot 7 \dots \cdot (2n+1)}$$

and for even powers of cosine

$$\int_0^{\pi/2} \cos^{2n} x \, dx = \frac{1 \cdot 3 \cdot 5 \dots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \dots \cdot (2n)} \frac{\pi}{2}$$