

**Homework 8**  
**Due 7/21/05**

**Problems 1-6:** VII.14.1, VII.14.2, VII.16.1, VII.17.2, VII.17.5, VII.18.2.

**Problem 7:** Suppose  $f: G \rightarrow \mathbb{C}$  is nonconstant and holomorphic, and let  $K$  be a compact subset of  $G$ . Suppose that  $|f|$  is constant on  $\partial K$ . Show that  $f$  must have at least one zero in  $K$ .

(Correction: we need to assume also that the interior of  $K$  is nonempty, or else the statement is trivially false.)

**Problem 8:** Let  $f$  be holomorphic in a region containing the closed unit disk. Suppose that  $f(-\ln 2) = 0$  and  $|f(z)| \leq |e^z|$  for all  $z$  on the unit circle. How large can  $|f(\ln 2)|$  be? Exhibit an  $f$  satisfying the conditions above such that  $|f(\ln 2)|$  is as large as it can possibly be. (This is a slightly extended version of a problem on the fall 1998 prelim exam at Berkeley. Hint: you'll need to use Schwarz's lemma somehow, and it will probably behoove you to think about some LFTs.)