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Math 185, summer 2005

Homework 7
Due 7/19/05

Problem 1: Let f and g be holomorphic functions. Show that the Taylor series of $h = fg$ equals the Cauchy product of the Taylor series for f and the Taylor series for g. (It will certainly help to read VII.9; you're just being asked to fill in the details of the "standard induction argument" referred to there.)

Problems 2-7: VII.8.1, VII.8.2, VII.9.2, VII.10.1, VII.11.1, VII.11.2.

Problem 8: (Lagrange interpolation–this might be easy if you've already done it in 128A!) Suppose we are given two lists of numbers $z_1, z_2, \ldots, z_n \in \mathbb{C}$ and $a_1, a_2, \ldots, a_n \in \mathbb{C}$. Suppose that the z_i are distinct. Show that there is a unique polynomial P of degree $< n$ such that $P(z_i) = a_i$ for $1 \leq i \leq n$.

(Big hint: consider the polynomial

$$\sum_{k=1}^{n} \frac{a_k Q_k(z)}{Q'(z_k)}$$

where $Q(z) = \prod_{j=1}^{n}(z - z_j)$ and $Q_k(z) = Q(z)/(z - z_k)$. To get uniqueness, you should probably show that a polynomial of degree ℓ has at most ℓ roots.)

(b) Just to make sure you understand what's going on, find the unique polynomial of degree < 3 such that $P(1) = 3$, $P(-1) = 1$, $P(2) = 7$, using the recipe above. (There are other ways to do part (a), I suppose–if you've heard of van der Monde matrices, then you can probably see a proof based on those–but the hint is nice because it constructs P explicitly.)