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Math 185, summer 2005

Homework 5
Due 7/12/05

Problems 1-5: V.12.1, V.12.2, V.14.1, V.14.2, V.17.1.

Problem 6: Recall that a series $\sum_{n=1}^{\infty} g_n(z)$ of functions is said to be uniformly convergent if the sequence of partial sums is uniformly convergent. Prove the following result:

Theorem: ("Weierstrass M-test") Let (g_n) be a sequence of functions on a set $G \subseteq \mathbb{C}$. Suppose that there is a sequence of real constants $M_n \geq 0$ such that

(1) $|g_n(z)| \leq M_n$ for all $z \in G$
(2) $\sum_{n=1}^{\infty} M_n$ converges

Then $\sum_{n=1}^{\infty} g_n(z)$ converges absolutely and uniformly on G.
(Hint: Use exercise V.6.1.)

Problem 7: Show that if the sequence of functions (g_n) converges uniformly to g on a set G, and the g_n are all continuous on G, then g is continuous on G. (Later we will prove a similar result about analytic functions.)