Patrick Corn
Math 185, summer 2005

Homework 4
Due 7/5/05

Problem 1: We say that the points z and w are symmetric with respect to the clircle $C \in \overline{\mathbb{C}}$ if there is an LFT ϕ such that $\phi(C)$ is the real axis and $\overline{\phi(z)} = \phi(w)$.

(a) Show that if z and w are symmetric with respect to C, then *any* LFT ψ sending C to the real axis satisfies $\overline{\psi(z)} = \psi(w)$. So we can loosely say that the definition of symmetry is "independent of the LFT."

(b) Show that if z and w are symmetric with respect to C and ϕ is an LFT, then $\phi(z)$ and $\phi(w)$ are symmetric with respect to $\phi(C)$. This is called the symmetry principle.

(c) Let C be a circle, with center a. Show that a and ∞ are symmetric with respect to C. (Hint: Maybe you should do it first when C is the unit circle. Then use the symmetry principle.)

(d) Find the LFT ϕ sending $|z| < 1$ to $|z-1| < 1$ such that $\phi(-1) = 0$ and $\phi(0) = 2i$. (Hint: Of the many variously difficult ways to do this, one stands out: use the symmetry principle to figure out $\phi(\infty)$. You will still have to do a bit of computation.)

(e) Show that if z and w are symmetric with respect to a circle centered at a, then the three points a, z, w are collinear, and the geometric mean of $|z-a|$ and $|w-a|$ is the radius of the circle. (Hint: try to show that $(z-a)(\overline{w}-\overline{a}) = R^2$, where R is the radius.)

Problem 2: Exercises IV.13.3 and IV.13.4 (once you do the first, the second will follow right away).

Problem 3: Find all possible values of $(1+i)^{1+i}$.

Problems 4-6: IV.13.1, V.6.1, V.6.2.