

Homework 2
Due 6/28/05

Problem 1: Suppose U is an open subset of \mathbb{C} with the following property: given any two points $a, b \in U$, there is a differentiable path in U from a to b , that is, a map $\gamma: [0, 1] \rightarrow U$ with $\gamma(0) = a$ and $\gamma(1) = b$. Prove that if $f: U \rightarrow \mathbb{C}$ is holomorphic with $f'(z) = 0$ for all $z \in U$, then f is constant. Show that the hypothesis on U is necessary by exhibiting an open subset V of \mathbb{C} lacking this property and a holomorphic non-constant function $f: V \rightarrow \mathbb{C}$ whose derivative is everywhere zero.

(Extra credit: show that the property is equivalent to connectedness, as defined in class.)

Problems 2-9: From the text: II.6.2, II.6.3, II.8.1, II.8.2, II.15.5, III.9.3, III.9.5, III.9.6.

Problem 10: Let $f: G \rightarrow \mathbb{C}$ be a continuous function (where, as usual, G is an open subset of \mathbb{C}). Suppose that $f(a) \neq 0$ for some $a \in G$. Show that there is a neighborhood N of a contained in G such that $f(z) \neq 0$ for all $z \in N$.