Real-p-Adic Analysis

Lecture 29

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Dimension

Relative Interior

Suppose $\phi: Y = \mathcal{M}(B) \to X = \mathcal{M}(A)$. $Int(Y/X)$ is the set of $y \in Y$ such that there is a bounded admissible surjection $f: A_r\langle T\rangle \to B$ such that $|f(T_i)|_y < r_i$ and $\partial(Y/X) := Y \backslash Int(Y/X)$.

Lemma. *If* $Z = \mathcal{M}(C)$, $Int(Y \times Z) = Int(Y) \times Int(Z)$ *and so*

$$\partial(Y \times Z) = (\partial Y \times Z) \cup (Y \times \partial Z)$$

Lemma. $\partial K = \emptyset$ *and* $\partial \mathcal{M}(K\langle T\rangle) = \{$ *Gauss norm*$\}$.

Proof. Let $I_y := \{f \in R\langle T\rangle : |f|_y < 1\}$. Then $I_y = (\pi, g(T))$ where $g \in R[T]$, $g \bmod \pi$ is prime or zero.

Suppose $\deg g = n$, g is monic, $|g(T)|_y < \epsilon < 1$ and $q > 1/\epsilon$. Define $f: K_{(q,1,\ldots,1)}\langle T_o, T_1, \ldots, T_n\rangle \to K\langle T\rangle$ by $f(T_0) = T$ and $f(T_i) = T^{i-1}g(T)$, $n \geq i \geq 1$.

Theorem. *If* Y *is a baffinoid domain in* X, *then* $Int(Y/X)$ *equals the topological interior of* Y *in* X.

Dimension

If $\mathcal{C}$ is a collection of subsets, its order is the largest integer n such that $\mathcal{C}$ contains $n+1$ sets with non-empty intersection. The **dimension** of a normal topological

"It [is] pure coincidence that the real line coincides with the field of real numbers."

space X is the smallest integer n such that every cover has a refinement of order at most n.

Suppose that for any pair of disjoint closed subsets C, B, there exists an open U such that

$$C \subset U \subset \bar{U} \subset X \backslash B,$$

and $\dim(\bar{U} \backslash U) \leq n-1$. Then $\dim X \leq n$.

Lemma. *Suppose B, C are disjoint closed subsets of an affinoid. Then there exists a special nbhd V of C and $V \cap B = \emptyset$.*

Proposition. *If $X = \mathcal{M}(A)$ is affinoid,* $\dim X = \dim A$ *and and*

$$\dim \partial \mathcal{M}(V) = (\dim V) - 1.$$

Suppose $A_n(K) = K\langle T_1, \ldots, T_n\rangle$ and $M_n(K) = \mathcal{M}(A_n(K))$. We already know $\dim M_n(K) \geq n$.

Lemma. *Suppose $\phi\colon Y \to X$ is finite surjective and Y and X are pure. Then* $\dim X = \dim Y$. *If X and Y are baffinoid, then $\partial Y = \phi^{-1} \partial X$.*

Example. We have $\phi_i\colon M_n \to M_1$ from $T \mapsto T_i$.

This implies $\dim X \geq \dim A$.

Proof of Proposition by induction

Its true in dimension 0.

Suppose $\dim A = n$ and $\alpha\colon A_n \hookrightarrow A$. Claim: $\partial X = (\alpha^*)^{-1} \bigcup_i \phi_i^{-1} \partial M_1$ and $\dim \partial X = n-1$.

Suppose V is a special nbhd of C and $V \cap B = \emptyset$. Then $\dim \partial(V/X) \subseteq \partial V$ so $\dim \partial(V/X) \leq n-1$.

Set $U = Int(V/X)$. $\dim \partial(U/X) \leq n-1$.