Galois Theory

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Lecture 8

K-homomorphisms

$x^p - x - 1$ is irreducible over $\mathbf{F}_p$ (again).

Theorem. *If C/K is algebraically closed and L is a separable extension of K of degree n there exist exacly n distinct K-homomorphisms from L into C.*

Proposition. *Suppose E is an extension of K. Then if a and b are in E and algebraic over K, there exists a K-homomorphism $\sigma\colon K(a) \to K(b)$ such that $\sigma(a) = b$ if and only if a and b are conjugate. Moreover, if σ exists it is unique and an isomorphism.*

Proof.

Lemma. *Suppose C is algebraically closed $\sigma\colon L \to C$ is a homomorphism and E is a seperable extension of L of degree n such that $E = L[\alpha]$. Then there exist exactl'y n extensions of σ to E.*

Proof.

Proof of theorem.

Non-*Example.*

Primitive Element Theorem

Theorem. *Let L be a finite separable element of K. Then there exists $\alpha \in L$ such that $l = K(\alpha)$.*

We'll prove his when K is infinite.

Lemma. *If V is a vector space over K, V is not the union of finitely many proper subspaces.*

Proof.

Proof of theorem.

Non-*Example.*

Homework for Monday

Read §6.5. Do 6.2 (Regard the "solutions" in the book as hints.)