Galois Theory

Robert F. Coleman

Lecture 29

Finite Fields

Theorem. *Let p be a prime and r a posiive integer. Then, up isomorphism, there exists one field of order p^r which we call $\mathbf{F}_{p^r}$. It is a splitting field over $\mathbf{F}_p$ of $X^{p^r} - X$.*

Theorem. *If $s = dr$, $\mathbf{F}_{p^s}$ is a normal seperable extension of $\mathbf{F}_{p^r}$ of degree d and $Gal(\mathbf{F}_{p^s}/\mathbf{F}_{p^r})$ is cyclic generated by $\phi\colon x \mapsto x^{p^r}$.*

Lemma. *If K is a field of characteristic p, $q = p^r$ then, $\{x \in K : x^q = x\}$ is subfield.*

Proof.

Proof of Theorems.

Sanity Check

Let $F \subset E \subset K$ be subfields of $\mathbf{C}$. Let $G \supset N \supset (e)$ be subgroups of G. Suppose K/F is normal, finite and $\mathrm{Gal}(K/F) = G$.

Which of the following are true?

A. K/E is normal. B. E/F is normal.

Suppose $E = K^N$ and N is normal. Then what are $\mathrm{Gal}(K/E)$ and $\mathrm{Gal}(E/F)$?

Now suppose L is a subfield of K, $F \subset L \subset E$ and $H = G_L$. Is $N \subset H$? What is K^H? When does $\operatorname{Gal}(E/L)$ make sense and what is it if it does?

Now suppose M is an extension of F in $\mathbf{C}$ and $P = K(\{m \in M\})$. Why is P normal over M?

What assumptions on M will make P finite and normal over F?

Read §14.1-§14.4

Homework for Monday

Do 14.4, 14.6.