Galois Theory

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Lecture 11

Mid-Term; Friday, March 2

Normal Extensions

An algebraic extention L/K is **normal** if every irreducible polynomial over K with one root in L factors completely in L. Eg. If L is algbraically closed it is normal.

Suppose C/K is algebraically closed. We (almost) proved

Proposition. *An extension N of K contained in C is normal if and only if the image of every K-homomorphism $N \to C$ is contained in N.*

but we assumed

Lemma. *Suppose E/K is algebraic, L/K is contained in E ad $h: L \to c$ is a K-homomorphism. then h extends to E.*

You might get to prove this on the midterm.

Suppose $P(x)$ is a monic polynomial over K, L is an extension of K and $\alpha_1, \ldots, \alpha_n \in L$ such that $L = K(\alpha_1, \ldots, \alpha_n)$ and $P(x) = \prod_i (x - \alpha_i)$. Then L is called a splitting field of P over K.

Proposition. *Splittng fields over K in C are normal.*

Proof.

Proposition. *Suppose $P(x)$ is a polynomial over K of degree n and L is a splitting field of P. Then $[L : K] | n!$.*

Proof.

Lemma. *Finite separable normal extensions are splitting fields.*

Proof.

Normal Closure

If L/K is an extension, a normal closure of L is a minimal normal extension containing L.

Read §7.5-§7.7.

Homework for Wednesday

Do 7.2 and 7.3b (use Viete).

Possible Midterm Problems

1. Prove every monic polynomial has a splitting field.
2. Let $g(E) = E^6 + 2E^3 - 1$ and K be a splitting field of K over $\mathbf{Q}$. List as many distinct subfields of K. Prove they are distinct.
3. Suppose C/K is algebraically closed. Suppose E/K is algebraic, L/K is contained in E ad $h\colon L \to c$ is a K-homomorphism. then h extends to E.