Algebraic Number Theory

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Lecture 18

Fredholm Theory

For $u = (u_1, \ldots, u_n) \in \mathbf{N}^n$, let $|u| = \sum u_j$, an Let $S_{n,M}$ be

$$\{G \in K[[X_1, \ldots, X_n]]\colon G = \sum_{w \in \mathbf{N}^n} g_w X^w,$$
$$\mathrm{ord}_p g_w - M|w| \to \infty \quad \text{as } |w| \to \infty\}.$$

These are the series which converge on the polydisk

$$D_M = \{(x_1, \ldots, x_n)\ \mathrm{ord}_p X_i \geq -M\}.$$

Set $||G|| = \max\{p^{M|w| - \mathrm{ord}_p g_w}\}$. Then

$$||G|| = \max_{x \in D_M(\mathbf{C}_p)} |G(x)|.$$

Suppose π_m is the projection of $S =: S_{n,M}$ onto $S_m =: \mathrm{span}\{X^w\colon |w| \leq m\}$. Then we say a linear map L from $S_{n,N}$ to S is **completely continuous** if

$$\lim_{m \to \infty} \pi_m \circ L \to L.$$

Lemma. *If L is completely continuous and F is continuous, $L \circ F$ and $F \circ L$ are completely continuous.*

Lemma. *If $N > M$ the restriction map is ompletely continuos.*

Proof.

Corollary. ψ_q *is completely continuous.*

Lemma. m_G *is continuous.*

$\det(1 - \pi_m \circ L|_{S_m} T)$ converges.