Linear Algebra

Transpose

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Let $\hat{V} = \mathcal{L}(V)$. If $L \in \mathcal{L}(V)$, define $\hat{L} \in \mathcal{L}(\hat{V})$ by $\hat{L}(f)(v) = f(L(v))$. Then

$$\delta_i(\hat{L}) = \widehat{\delta_i(L)}.$$

If $v_1, \ldots, v_n$ is a basis for V and $\hat{v}_i$ is the element of $\hat{V}$ such that $\hat{v}_i(v_j) = \delta_{i,j}$ then $\hat{v}_1, \ldots, \hat{v}_n$ is a basis for $\hat{V}$.

If M is the matrix for L with respect to $v_1, \ldots, v_n$ then M^T is the matrix for $\hat{L}$ with respect to $\hat{v}_1, \ldots, \hat{v}_n$.

Now conclude $\det M = \det M^T$.