Linear Algebra

Lecture 9

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A Reconciliation

From now on, if U and V are subspaces of a vector space W, we will say W is **the direct sum** of U and V if the map $(u, v) \mapsto u + v$ from $U \oplus V \to W$ is an isomorphism.

Exercises

8. Find a basis for $\{(x_1, x_2, x_3, x_4, x_5)\colon x_1 = 3x_2, x_3 = 7x_4\}$.

11. Show if $U \subseteq V$, $\dim U = \dim V$ then $U = V$.

12. Show $p_0, \ldots, p_m$ are dependent if $p_j(2) = 0$ for all j and all p_j have degree at most m.

13. Suppose $\dim U = 5$ and $\dim V = 3$. show $U \cap V = 0$ if and only if $U + V = \mathbf{R}^8$.

16. Show $\dim(U_1 + U_2 + \cdots + U_n) \leq \dim U_1 + \dim U_2 + \cdots + \dim U_n$.

17. Show $\dim(U_1 \oplus U_2 \oplus \cdots \oplus U_n) = \dim U_1 + \dim U_2 + \cdots + \dim U_n$.

Chapter 3

1. Show every linear map from a one-dimensional space to itself is multiplication by a scalar.

3. Every linear map from a subspace U of V can be extended.

5. If T is injective and $v_1, \ldots, v_n$ are LI then so are $Tv_1, \ldots, Tv_n$.

2. Null Space and Range

Example. Let V be the twice differentiable real valued functions on $\mathbf{R}$ and W the space of real valued functions on $\mathbf{R}$. Let $L(f) = f'' + f$.

Proposition. *Suppose $L: V \to W$ is a linear map. Then L is injective if and only if null $L = \{0\}$.*

Rank plus Nullity Theorem. *Suppose V is finite dimensional and $L: V \to W$ is a linear map. Then range L is finite dimensional and*

$$\dim V = \dim(\operatorname{null} L) + \dim(\operatorname{range} L).$$

Read Chapter 4. In Chapter 3 do problems 16. 19. 22. and 26.