Linear Algebra

Lecture 6

Robert F. Coleman

1. Review

If V is spanned by m independent vectors, any linearly independent subset of V has at most m elements and any subset with more than m elements is dependent. A set of vectors S in V is a basis if and only if the elements in S are independent and $\text{span}(S) = V$. When V has a finite basis with n elememts we say V has dimensxion n.

2. Dimension

Theorem. *(i) Every finite spanning set can be reduced (by removing vectors) to a basis and (ii) every linearly independent set in a finite dimensional vector space V can be extended (by adding vectors) to a basis.*

Proof. First (i) is easy. Now if V is finite dimensional, we know, there is an $m \geq 0$ so that any linearly independent subset of V has fewer than m elements. Suppose $(v_1, \ldots, v_n)$ is a linearly independent n-tuple (or list).

Corollary. *If V has dimension n any subset of n independent elements is a basis.*

Proof.

The space of solutions of the differential equation $f'' + f = 0$ has dimension 2.

Because we can always extend an independent set of vectors to a basis we get,

Lemma. *If U is a subspace of V, $\dim U \leq \dim V$.*

3. Linear Dynamism

Suppose V and W are vector spaces over $\mathbf{F}$. A linear map f from V to W is a function $f\colon V \to W$ that preserves linear structure, i.e. is such that

$$f(av + bw) = af(v) + bf(w).$$

Examples.

(i) Scalar Multiplicattiion.

(ii) Evaluation.

(ii) Differentiation

(iv) Inclusion

(v) Polynomials

For next time: Read pages 37-47(again). Do problems 13, 16, 17 in Chapter 2..