Linear Algebra

Lecture 2

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1. The reals and complexes

The symbol $\mathbf{F}$ will denote either the real numbers $\mathbf{R}$ or complex numbers $\mathbf{C}$ throughout this course. They are both examples of what are called fields. These are sets with two operations, addition and multiplication. Each is commutative and associative, and multiplication distributes over addition. There is an additive identity, usually called 0, additive inverses, a multiplicative identity, usually called 1, and non-zero elements have multiplicative inverses.

Any element of $\mathbf{C}$ can be written in the form $a + bi$ where a and b are real numbers and $i^2 = -1$.

Example. Suppose a and b are real numbers.

$$(a + bi)(a - bi) =$$

Other examples:

2. Vector Spaces

A vector space over $\mathbf{F}$ is a set with a commutative, associative addition, an additive identity also denoted 0, additive inverses and an associative multiplication by $\mathbf{F}$ which distributes over addition (in both senses) such that $1 \cdot v = v$.

Then $\mathbf{F}$ is a vector space over itself and $\mathbf{C}$ is a vector space over $\mathbf{R}$. Other examples include, the set of n-tuples of elements of $\mathbf{F}$, $\mathbf{F}^n$, and continuous real valued functions on the real line.

There are a number of basic properties of vector spaces which follow from the definition; including, uniqueness of the additive identity, uniqueness of additive inverses, $0v = a0 = 0$ and $-1v$ is the additive inverse to v.

Proof that $-1 \cdot v$ is the additive inverse to v.

3. Subspaces

Suppose V is a vector space over $\mathbf{F}$. Then a subset W of V is called a **subspace** if it is also a vector space using the addition of V and the multiplication of $\mathbf{F}$ on V.

Proposition. *W is a subspace of V if and only if it is non-empty, and closed under addition and scalar multiplication.*

Discussion.

Example. The set of differentiable functions on $\mathbf{R}$ is a subspace of the set of continuous functions.

Every vector space has a subspace.

For next time: Read pages 11-16b. Do problems 6, 8 and 9 in Chapter 1.