Linear Algebra

Lecture 1

Robert F. Coleman

First of all, why is this course called Linear Algebra? Linear Algebra is the mathematics associated with linear equations, alternatively, it is the algebra needed to understand the geomtry of lines, planes and their higher dimensional generalizations. It is the algebra of "first approximations." In general, things in the real world are not linear but if they were they would be a lot simpler. The geometry of lines and planes is much easier to understand than the geometry of ellipses and ellipsoids. As you learned in calculus when you look real close things look linear; the derivative is the slope of the tangent line at a point. In this course, we will begin the systematic study of all things linear. Once you know how lines work you can start tampering with more complicated things, like groups, algebraic geometry, differential equations and physics.

I will assign three or four problems per class due the next Friday. We will discuss solutions in class. I will be encourage you to discuss your solutions and/or attempts at solutions. This is one way I can get to know you. Others include office hours and email. Ask questions. Hand a question on Monday each week. (The more questions you ask me, the fewer I'll ask you.) I certainly won't be able to cover in class, all the material in the book, you'll be required to learn. As a result you will be expected to do a lot of reading. I'll put the reading and homework assignments at the end of the lecture notes. There will be one midterm and one final (and several quizes). I will also put course notes etc. on the web at
www.math.berkeley.edu/˜coleman/Courses/Spring07/linear.html

Grading: Homework 30%, Midterm+Quizes 30%, Final 40%. Office hours: 901 Evans, (this week) Monday 3-4, Weds. 2-3. Email: coleman@math.berkeley.edu

Our GSI is Patrick borisp@Math.Berkeley.EDU who will help you ask questions and me answer them. The text is Linear Algebra Done Right (2nd ed) (= Linear Algebra in paperback).

Vector Spaces. The fundamental problem in linear algebra is: How can you tell if there are there solutions to a set of equations like,

$$a_{12}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

where the a_{ij} and the b_i are in **C** (the complex numbers) or **R** (the real numbers) and the x_i are variables and how do you find them?. Eg., Find all real numbers x_1, x_2, x_3 such that

$$x_1 + 2x_2 + 3x_3 = 0 \quad - x_1 + 4x_2 + 2x_3 = 1 \quad 6x_2 + 5x_3 = 1.$$

Now let me start saying things that you may not completely grasp yet but should by the end of the course.

We get to rewrite the above equatrions as

$$MX = B, \tag{$*$}$$

where M is the matrix $\begin{pmatrix} 1 & 2 & 3 \\ -1 & 4 & 2 \\ 0 & 6 & 5 \end{pmatrix}$ and, X and B are the column vectors, $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$. A solution exists because the second row is not a multiple of the first, the third row is the sum of the first two and $0 + 1 = 1$.

Now if X is a solution and Y is a column vector such that $MY = 0$ (the zero column vector), $X + Y$ is another solution and if X and X' are two solutions

$M(X - X') = 0$. Thus, it seems like a good idea to consider all solutions of the equation

$$MY = 0. \tag{$**$}$$

Indeed, if you have one solution of $(*)$ and all solutions of $(**)$ then you'll have all solutions of $(*)$. The set V of solutions of $(**)$ is a vector space. This means that any linear combination of elements of V is an element of V. It turns out that, in this case, if we have one solution X of $(*)$ one non-zero solution Y of $(**)$ all other solutions of $(*)$ are of the form $X + cY$, for a real number c. So to solve our original set of equations we just need one solution of it and one non-zero solution of $(**)$.

Problems for Friday and reading for next time: Read pages 1-10. Do exercises 2 and 4 of Chapter 1.