Linear Algebra

Lecture 13

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1. Eigenvalues and Eigenvectors

Let $V = \mathbf{F}_3[T]$ and $L(g)(T) = \frac{\mathrm{d}}{\mathrm{d}T}((T+1)g(T))$.

$$L(T) = 2T + 1,\ L^2(T) = 4T + 3$$

$3L(T) - 2T = L^2(T)$.

2. Questions

When we assign a linear map $T(v_i) = w_i$, can the w_i be anything?

Would it be all useful to extend rank-nullity to transfinite dimensional vector space?

If T is injective can we can make it an isomorphism by changing the codomain?

What use is $P(T)$ for $T \in \mathcal{L}(V)$?

An example.

Let L be the operator on $\mathbf{F}^2$ whose matrix (with respect to the standard basis) is

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Homework:

A. Find the other eigenvalues for the operator in §1 above.

B. An operator N on a vector space is said to be nilpotent if $N^n = 0$. Show if V is finite dimensional and N is nilpotent, the matrix of N with respect to some basis of V is upper triangular with zeroes on the diagonal.