Ordinary Differential Equations

Robert F. Coleman

Office: 901 Evans.

Email address: coleman@math.berkeley.edu

Website: http://math.berkeley.edu/˜coleman/Fall05/ode.html

Office hours: Today.2:10-3, Fri. 3:10-4. (or to be arranged by email)

Each class, I will give reading and homework assignments, due by the next class, of about 3 problems each of which 1 or 2 random problems will be graded. Also, each week, on Monday, you must hand in at least one questions about the reading or problems (which, I'll try to answer). There will be one midterm. Student presentations will be encouraged.

Grading: Questions & Homework 30%, Midterm 30%, Final 40%

Text: Ordinary Differential Equations and Stability Theory by David Sanchez

Examples. 1. Suppose you know that the number of bacteria in your cut increases more rapidly the more bacteria there are in your cut. How many bacteria will there be after one hour?

Let $B(t)$ be the number of bacteria at time t suppose there are 10 bacteria at 12AM and there are $h+1$ times as many bacteria after h seconds. I.e.,

$$\frac{B(t+h)-B(t)}{h} = B(t).$$

Assume $B(t)$ is a smooth function of time and forget that h is an integer. Then we get $B'(t) = B(t)$. Thus,

$$B(t) = 10 \cdot \exp(t).$$

After one hour the number of bacteria will be

$$\exp(60 \cdot 60) = 2.884927155669398615533200926 \cdot 10^{1563}.$$

2. Suppose $v(t)$ is the velocity of a partical moving along the x-axis at time t. Suppose at time 0 it is at x_0. Where will it be at time t?

Let $x(t)$ be its position at time t. We know $x(0) = x_0$ and

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = v(t).$$

Thus,

$$x(t) = x_0 + \int_0^t v(x)\mathrm{d}x.$$

By an ordinary differential equation I mean an equation of the form

$$F(x, y, y', \ldots, y^{(n)}) = 0.$$

where y is a function of x. Eg.,

3. The van Der Pol Oscillator is the first relaxation oscillator. van Der Pol described this oscillator in a paper in 1928 in a model of human heartbeat. It is also describes the behavior of a circuit with a tunnel diode.

Here is the second-order differential equation for a van Der Pol oscillator:

$$v'' - \alpha(1 - v^2)v' + \omega^2 v = 0.$$

www.iro.umontreal.ca/ eckdoug/vibe/Relaxation/VanDerPol.html For these we need **initial conditions**. You can think about the equation as a relation the function, its rate of change and rate of rate of change etc. satisfies.

First Order Equations

A first order equation is one of the form

$$M(x, y) + N(x, y)y' = 0$$

Eg. (1) $x' = y$ and (2) $y'y + x = 0$. Since $\mathrm{d}(x^2 + y^2) = 2x\mathrm{d}x + 2y\mathrm{d}y$, if $x^2 + y^2$ is a constant, y is a solution of (2). Suppose we want $y(0) = C^2$. Then $y(x) = \sqrt{C^2 - x^2}$ is our solution.

For Weds.

Read 1-4m, Do problem 2 of chapter 1.