

H54 Midterm

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Do all three problems. Justify your answers. Hand in your exam in section on Friday December 2.

1. Suppose $M = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. What is the smallest $\|Mw - v\|$ can be for $w \in \mathbf{R}^3$? How many such w are there?
2. Suppose you have a 10cm bar of silver and at time 0 you heat half of it to 100 degrees while keeping the other half at zero degrees. Then you remove the heat source and insulate both ends. Will the point 2.5 cm from the end on the unheated side be warmer after 3 seconds than after 5 seconds?
3. Suppose $Q(x) = (ax + by + cz)^2$ where $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Find real numbers $d \geq e \geq f$ so that there exists an orthogonal matrix L so that $Q(Lx) = dx^2 + ey^2 + fz^2$.