Linear Algebra and Differential Equations

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Lecture 9

1. Tigers and Zebras

Suppose $Z(x)$ is the number of zebras and $T(x)$ is the number of tigers at time x. Suppose every pair of zebras gives birth eight more and every tiger eats one. I.e.,

$$Z'(x) = 4Z(x) - T(x)$$

On the other hand suppose every pair of tigers gives birth two plus two times the number of Zebras they could eat more. I.e.,

$$T'(x) = T(x) + 2Z(x).$$

Find $Z(0)$ and $T(0)$ so that $Z(x)/T(x)$ is constant.

$$\begin{pmatrix} Z \\ T \end{pmatrix}' = \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} Z \\ T \end{pmatrix}$$

Ordinary Differential Equations

These are equations of the form

$$x'(t) = F(t, x(t))$$

If x is a real valued function, one say s this equation is of the **first order.** *Examples.*

Fundmental Theorem of Calculus. *If $g(t)$ is continous on the interval $[a, b]$ and $c \in \mathbf{R}$ there is one and only one solution $y_c(t)$ to the equation*

$$y' = g(t)$$

such that $y_c(a) = c$. *It is*

$$c + \int_a^t g(x)\mathrm{d}x$$

Note that $y_{c+d} = d + y_c$.

Population Growth $x' = x^2$

Suppose $x(0) = p$. Then

$$x(t) = \frac{p}{1 - pt}.$$

Separation of variables

$x' = f(t)/g(x)$. Think of this as $g(x)\mathrm{d}x = f(t)\mathrm{d}t$. Supppose you want $x(t_0) = x_0$.

Eg. $tx' + x = x^2$.

Existence and Uniqueness

Suppose F_i is a real valued function of t and $x_1, \ldots, x_n$. Consider the system

$$x_i' = F_i(t, x_1, \ldots, x_n)$$

EU Theorem. *If the* F_i *have continous derivatives in the* x_i *at the point* $(t, x_1, \ldots, x_n) = (t_0, a_1, \ldots, a_n)$ *. Then a solution* $\mathbf{x} = (x_1(t), \ldots, x_n(t))$ *of the above system exists satisfying*

$$(x_1(t_0), \ldots, x_n(t_0)) = (a_1, \ldots, a_n)$$

on some interval containing t_0. *Moreover, any other solution satisfying this condition agrees with this one on any interval where they are both defined.*

Example. $x' = x^{3/2}$

Homework for Wednesday

Read pages 36-40m Do Exercises 2.1.1 (a), (b) and (c).