Linear Algebra and Differential Equations

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Lecture 5

Matrices and matrix operatsions

We have

$$MV := \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

We can write this as

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} V = \begin{pmatrix} \langle R_1, V \rangle \\ \langle R_2, V \rangle \end{pmatrix}$$

Quadratic Forms and Transpsoses

$$\langle V, MV \rangle = x(ax + by) + y(cx + dy) = ax^2 + (b + c)xy + dy^2$$

$$F(V, W) = \langle V, MW \rangle$$

is bilinear.

Lemma. *If* $\langle u, v \rangle = \langle w, v \rangle$ *for all vectors* v, $u = w$.

Proof.

Proposition. *There is a unique matrix* M^t *such that*

$$\langle V, MW \rangle = \langle M^t V, W \rangle$$

for all vectors V *and* W. *in fact,*

$$M^t = \begin{pmatrix} a & c \\ b & d \end{pmatrix}.$$

Proof.

Lemma. $(AB)^t = B^t A^t$.

Proof.

Lemma. *The following are equivalent:*

(i) $|MV| = |V|$ *for all* V

(ii) $\langle MV, MW\rangle = \langle V, W\rangle$ *for all* V *and* W,

(iii) $M^t M = I$.

Such a matrix is called **orthogonal**.

Proof.

Homework for Monday

Read 18m-20m. Do exercises 1.3.4 (a), (c), (d) and (i).