Linear Algebra and Differential Equations

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Lecture 2

Volterra Equation of the second kind. *Given a continuous function* g *on the interval* $[0,1]$, *find* f *such that*

$$f(x) = \int_0^x f(y)K(x,y)dy + g(x).$$

A **vector** is an object with length and direction. Given two points A and B in the plane, we get a directed segment $\vec{AB}$ from A to B. This represents a vector. $\vec{CD}$ reprsesents the same vector if the lines through the segments are parallel and the lengths are equal.

One can add vectors and multiply them by scalars.

The inner product

$$\langle u, v\rangle = |u||v|\cos\theta$$

Clearly, $\langle u, v\rangle = \langle v, u\rangle$ and $\langle u, u\rangle = |u|^2 \geq 0$.

We will prove $\langle\ ,\ \rangle$ is bilinear.

Example. Suppose A, B and C are points in the plane. If $\vec{AB}$ represents v and $\vec{BC}$ represents u,

$$|\ \vec{BC}\ |^2 = \langle v-u, v-u\rangle =$$

Coordinates

Given a point O which well call the origin and two directed perpindiclar lines through O we specify any point P in the plane by a pair (a, b) of real numbers. We call them the coordinates of the vexctor represente d by $\vec{OP}$. We denote this vector by $\binom{a}{b}$. then if $\mathbf{u} = \binom{a_1}{b_1}$ and $\mathbf{v} = \binom{a_2}{b_2}$

$$\mathbf{u} + \mathbf{v} = \binom{a_1 + a_2}{b_1 + b_2}, \quad \alpha\mathbf{u} = \binom{\alpha a_1}{\alpha b_1} \quad \text{and} \quad \big(\mathbf{u}, \mathbf{v}\big) = a_1 a_2 + b_1 b_2$$

Now we can check the bilinearity of (,).

$\mathbf{e_1} =: \binom{1}{0}$ and $\mathbf{e_2} =: \binom{0}{1}$ form a **basis**.

For Friday read sections 1.1.2 and 1.1.3 do exercises 1.1.2 (a) and (b) 1.1.3 (a) and (c).