Math Jams Worksheet 3

February 25th, 2008

Linear Maps and Polynomials

1. Let $V = P_{10}(\mathbb{R})$, and let a and b be two distinct real numbers. What is the dimension of the subspace $W \subseteq V$ consisting of those polynomial functions with roots at both a and b?

2. Let $V = P_{10}(\mathbb{R})$ and let $a \in \mathbb{R}$ be a real number. Let $d : P(\mathbb{R}) \to P(\mathbb{R})$ be the differentiation map, and let $\phi : P(\mathbb{R}) \to \mathbb{R}$ be the map sending $p(x) \mapsto p(a)$. Recall we showed both of these are linear maps. Describe the subspace $W = (\text{Null } \phi) \cap (\text{Null } \phi \circ d)$ in as much detail as you can.

Some Random Stuff

3. Show that if $T : V \to W$ is an injective linear map between finite-dimensional vector spaces, then there is a linear map $U : W \to V$ such that $U \circ T = 1_V$.

4. Let V be any finite-dimensional vector space, and let $V^* := \mathcal{L}(V, \mathbb{R})$ be the vector space of linear maps from V to $\mathbb{R}$ (called the *dual* of $\mathbb{R}$). Show that $V \cong V^*$.

5. Let V be a finite-dimensional vector space, and let $v \in V$ be an arbitrary vector. Show that the map $\phi_v : V^* \to \mathbb{R}$ defined by $T \mapsto T(v)$ is a linear map. Find the dimension of the null space of this map (in particular, does it depend on v?)

6. Continuing with the above, let $v_1, \ldots v_n$ be some n vectors in V, and let $\phi : V^* \to \mathbb{R}^n$ be the map defined by $T \mapsto (T(v_1), \ldots, T(v_n))$. What is the dimension of Null ϕ? When is this map injective?