# Math Jams Worksheet 2

February 25th, 2008

### More Linear Maps

1. Let $P_n(\mathbb{R})$ be the real vector space of polynomials of degree less than or equal to $n$. Let $\phi : P_n(\mathbb{R}) \to \mathbb{C}$ be the map sending $p(x) \mapsto p(i)$, where $i \in \mathbb{C}$ is the complex square root of $-1$. Give $\mathbb{C}$ the usual structure of a real vector space. Show that $\phi$ is a linear map, and find the dimension of the null space and image of $\phi$.

2. We showed on worksheet 1 that the differentiation operator $d : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ is a linear map. Is $d$ injective? What is the null space of $d$?

3. Let $d : P_{10}(\mathbb{R}) \to P_9(\mathbb{R})$ be the differentiation map. Is $d$ injective? Surjective? Is there a linear map $T : P_9(\mathbb{R}) \to P_{10}(\mathbb{R})$ such that $d \circ T = I$, where $I$ is the identity on $P_9(\mathbb{R})$?

4. Generalize the end of the last exercise: if $T : V \to W$ is a surjective linear map between finite-dimensional vector spaces, show that there is a linear map $U : W \to V$ such that $T \circ U = 1_V$, where $1_V$ is the identity on $V$. Is there a similar statement you can make when $T$ is injective?

### Eigenvectors and Eigenvalues 1

1. We showed on worksheet 1 that the differentiation operator $d : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ is a linear map. Find all eigenvectors for $d$.

2. Give an example of an invertible $3 \times 3$ matrix $A$ with real coefficients such that any two eigenvectors of $A$ are linearly dependent.