A function $f : \mathbb{Z} \to \mathbb{Z}$ is called *bounded above* if there is an $N \in \mathbb{Z}$ such that $f(m) \leq N$ for all $m \in \mathbb{Z}$.

1. Write the above definition using quantifiers.

   Solution: $\exists N \in \mathbb{Z}, \forall m \in \mathbb{Z}, f(m) \leq N$.

2. Using quantifier negation, write what it means to say that $f : \mathbb{Z} \to \mathbb{Z}$ is *not* bounded above. Rewrite your answer in English.

   Solution: $\forall N \in \mathbb{Z}, \exists m \in \mathbb{Z}, f(m) > N$. In English, this says that for any integer N, we can find an integer m such that $f(m) > N$.

3. Show that the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 2n$ is *not* bounded above.

   Solution 1: Use the definition from part (b). We want to show that $\forall N \in \mathbb{Z}, \exists m \in \mathbb{Z}, f(m) > N$. So, let $N \in \mathbb{Z}$ be arbitrary. If $N \leq 0$, then $f(1) = 2 > N$, so $m = 1$ works. If $N > 0$, then $f(N) = 2N > N$, so $m = N$ works.

   Solution 2: Use the definition from part (a), plus contradiction. Suppose that there exists an $N \in \mathbb{Z}$ such that $f(m) \leq N$ for all $m \in \mathbb{Z}$. If $N \leq 0$, then $f(1) = 2 > N$, a contradiction, so we must have $N > 0$. But then $f(N) = 2N > N$, again a contradiction. Hence our assumption was false, and no such N exists, so f is not bounded above.