Math 74 Midterm 2

November 12th, 2008

Name ______________________ SID _________________

Question	Score	Possible
1		9
2		7
3		7
4		7
$\sum$		30

1. Let X and Y be sets.

   (a) Show that $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$.

   (b) If $|X| = n$, $|Y| = m$, and $|X \cap Y| = \ell$, calculate $|\mathcal{P}(X) \cup \mathcal{P}(Y)|$.

   (c) With the same numbers as in (b), calculate $|\mathcal{P}_k(X \cup Y)|$, where $0 \leq k \leq |X \cup Y|$.

2. Let X be a set. Define a relation $\sim$ on $\mathcal{P}(X)$ by $A \sim B$ iff $A = B$ or $A = X \setminus B$.

   (a) Show that $\sim$ is an equivalence relation.

   (b) Let $x \in X$ be arbitrary. Show that the function

   $$f : \mathcal{P}(X)/\!\sim\, \longrightarrow \mathcal{P}(X)$$

   defined by

   $$f([A]) = \begin{cases} A & x \in A \\ X \setminus A & x \notin A \end{cases}$$

   is well-defined.

3. Let (X, d) be a metric space. Suppose that every bounded sequence in (X, d) converges. Show that $|X| \leq 1$.
   [Hint: Consider the sequence $x, y, x, y, x, y, \ldots$]

4. (a) Let $a, b, c \in \mathbb{N} \setminus \{0\}$. Show that if $\gcd(a, b) = 1 = \gcd(a, c)$, then $\gcd(a, bc) = 1$.

   (b) Let $a, b_1, \ldots, b_n \in \mathbb{N}\setminus\{0\}$ be arbitrary. Show that if $\gcd(a, b_i) = 1$ for all $i \in \{1, \ldots, n\}$, then $\gcd(a, b_1 \cdot b_2 \cdots b_n) = 1$.