

Math 74 Midterm 1
September 24th, 2008

Name _____ SID _____

Question	Score	Possible
1		5
2		10
3		8
4		9
Σ		32

1. Let $P, Q,$ and R be propositions. Let A be the proposition

$$P \Rightarrow ((Q \Rightarrow R) \text{ and } (R \Rightarrow Q)).$$

Let B be the proposition

$$(\text{not } P \text{ and } (Q \text{ or } R)) \text{ or } (P \text{ and } (Q \text{ and } R)).$$

Use truth tables to decide whether or not A and B are logically equivalent.

2. (a) Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function defined by $f(n) = 3n + 2$.

i. Is f injective? Prove your answer.

ii. Is f surjective? Prove your answer.

(b) Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function defined by $f(n) = \begin{cases} n & n \text{ even,} \\ \frac{n-1}{2} & n \text{ odd.} \end{cases}$

i. Is f injective? Prove your answer.

ii. Is f surjective? Prove your answer.

3. Let X and Y be sets, let $f : X \rightarrow Y$ be a function, and let $P(X)$ and $P(Y)$ denote the power sets of X and Y , respectively. Let $P(f) : P(X) \rightarrow P(Y)$ be the function defined by $P(f)(A) = \{f(a) \mid a \in A\}$.
- (a) Show that if $P(f)$ is injective, then f is injective. (Hint: think about one-element subsets of X).

- (b) Show that if f is injective, then $P(f)$ is injective.

4. Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be a function. We define f to be *eventually increasing* if for each $n \in \mathbb{Z}$ there is an $m \in \mathbb{Z}$ such that $m > n$ and $f(m) > f(n)$.

(a) Write the above definition using quantifiers.

(b) Again using quantifiers, write what it means for a function f to *not* be eventually increasing.

(c) Translate your answer from (b) into English.

(d) Define a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ by

$$f(n) = \begin{cases} 0 & n \text{ even,} \\ 1 & n \text{ odd.} \end{cases}$$

Prove that f is not eventually increasing.