Math 74 (Mini-)Homework 5
Due Monday, September 29th

September 25, 2008

1. Email me (at charleyc@math.berkeley.edu) with the following info:
   - Your enrollment status (enrolled or auditing).
   - What other math classes you're taking right now (if any).
   - Why you're interested in mathematics.
   - What you're hoping to learn from the rest of Math 74.
   - What are you planning on majoring in?
   - How do you feel about the class at the moment? What you would like to see done differently? What's working well for you?

2. The *division algorithm* for $\mathbb{N}$ says the following: if n and m are any two non-zero natural numbers, then there exist unique numbers $q, r \in \mathbb{N}$ such that $n = qm + r$ and $0 \leq r < m$. (You should think of this as saying that "m divides n q-times with remainder r.")

   (a) Fix an $m \in \mathbb{N} \setminus \{0\}$. Use the well-ordering principle to show that for every $n \in \mathbb{N} \setminus \{0\}$, there exist numbers $q, r \in \mathbb{N}$ such that $n = qm + r$ with $0 \leq r < m$.

   (b) Repeat the previous part of the problem, but give a proof using induction on n instead of the well-ordering principle.

   (c) Let $n, m \in \mathbb{N} \setminus \{0\}$ be arbitrary, and suppose that $q, q', r, r' \in \mathbb{N}$ such that $0 \leq r < m$, $0 \leq r' < m$, $n = qm + r$, and $n = q'm + r'$. Show that $q = q'$ and $r = r'$. (This is what was meant by "unique" in the statement of the division algorithm.)