

## Math 74 Homework 4

September 15, 2008

1. Do Eccles 8.2 and 9.1.
2. Which of the following functions are injective? Which are surjective? (You have to prove your answers.)
  - (a)  $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(n) = n^3$ .
  - (b)  $g : \mathbb{R} \rightarrow \mathbb{R}, g(n) = n^3$ .
  - (c)  $h : \mathbb{Z} \rightarrow \mathbb{N}, h(n) = |n|$ . (Here  $\mathbb{N}$  is the set  $\mathbb{N} = \{n \in \mathbb{Z} \mid n \geq 0\}$ .)
3. Let  $X, Y, Z$ , and  $W$  be sets, and let  $f : X \rightarrow Y, g : Y \rightarrow Z, h : Z \rightarrow W$  be functions. Prove that  $(h \circ g) \circ f = h \circ (g \circ f)$ . (One therefore says that “composition is an associative operation.”)
4. Let  $X, Y$ , and  $Z$  be sets, and let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be functions. Show that:
  - (a) If  $f$  and  $g$  are injective, then  $g \circ f$  is injective.
  - (b) If  $f$  and  $g$  are surjective, then  $g \circ f$  is surjective.
  - (c) If  $g \circ f$  is surjective, then  $g$  is surjective.
  - (d) If  $g \circ f$  is injective, then  $g$  is injective.
5. Let  $X, Y, Z$ , and  $W$  be sets, and let  $f : X \rightarrow Z$  and  $g : Y \rightarrow W$  be functions. Define a function  $(f \times g) : X \times Y \rightarrow Z \times W$  by  $(f \times g)(x, y) = (f(x), g(y))$  for all  $x \in X, y \in Y$ . Show that:
  - (a) If  $f$  and  $g$  are surjective, then so is  $f \times g$ .
  - (b) If  $f$  and  $g$  are injective, then so is  $f \times g$ .
  - (c) Let  $A$  and  $B$  be two more sets and let  $h : Z \rightarrow A$  and  $k : W \rightarrow B$  be two more functions. Show that  $(h \times k) \circ (f \times g) = (h \circ f) \times (k \circ g)$ .