Math 74 (Mini)-Homework 12
Due Monday, November 17th

November 15, 2008

Throughout, C is the set of Cauchy sequences in $\mathbb{Q}$ (with respect to the Euclidean metric), $\sim$ is the equivalence relation on C given by $(x_n) \sim (y_n)$ iff $\lim_{n\to\infty} x_n - y_n = 0$, and $R = C/\sim$.

1. Let $(x_n), (x'_n), (y_n)$, and (y'_n) be Cauchy sequences in $\mathbb{Q}$, and suppose $(x_n) \sim (x'_n)$ and $(y_n) \sim (y'_n)$. Show that $(x_n y_n) \sim (x'_n y'_n)$.

2. In class, we defined a relation $\leq$ on R by $[(x_n)] \leq [(y_n)]$ iff $(x_n) \sim (y_n)$ or there exists an $N \in \mathbb{N}$ such that for all $n \geq N$, $x_n \leq y_n$. We checked that this was well-defined. Show that it's a partial order relation.

3. Let $i : \mathbb{Q} \to R$ be the function which sends a rational number q to the class of the constant sequence (q) (i.e. the sequence $q, q, q, q, \ldots$). Show that:

   (a) The function i is injective.

   (b) For any $q, r \in \mathbb{Q}$, we have $q \leq r$ iff $i(q) \leq i(r)$.

   (c) Let d be the metric on R which we defined on Friday (we haven't checked that this is a metric yet). For any $q, r \in \mathbb{Q}$, show that $i(|q - r|) = d(i(q), i(r))$.