

Math 1A Worksheet 8

February 7th, 2008

1. Can the graph of a function have more than one tangent at a given point? If so, graph your answer; if not, explain why.
2. a) Evaluate $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$. This limit is the derivative of some function at some point; write the notation we typically use to describe this.
b) Similarly, evaluate $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x - a}$. Write your answer in “function notation” as in part a).
3. Let $f(x) = |x - 4|$.
 - a) Where is f continuous? Graph this function.
 - b) Find a formula for $f'(x)$ and sketch the graph of $f'(x)$. Is $f'(x)$ defined everywhere?

4. Let

$$f(x) = \left\{ \begin{array}{ll} 0 & , \quad x \text{ irrational} \\ 1 & , \quad x \text{ rational} \end{array} \right\}.$$

and let

$$g(x) = x^2 f(x).$$

Find $g'(0)$.

[Note: for those who know the “product rule” for derivatives: why does this rule **not** apply here?]

5. Give examples of functions which are:
 - a) Differentiable only at 0.
 - b) Continuous only at 0, but not differentiable at 0.
 - c) Continuous everywhere, and differentiable everywhere except -1 and 1 .
 - d) Differentiable only at -1 and 1 . [Hint for d): use a function like the one in Problem 2. You will probably have to do something piecewise.]