

# Math 1A Worksheet 7

February 5th, 2008

1. (From last time, if you didn't do it.) Let

$$f(x) = \begin{cases} 1 - x^2 & , \quad 0 \leq x \leq 1 \\ 1 + \frac{x}{2} & , \quad 1 < x \leq 2 \end{cases}$$

- a) Show that  $f$  is not continuous on  $[0, 2]$ .  
b) Show that  $f$  does not take on all values between  $f(0)$  and  $f(2)$ , that is, show that there is a number between  $f(0)$  and  $f(2)$  which is not a value of  $f$  on the interval  $[0, 2]$ .

2. Find

$$\lim_{x \rightarrow \infty} \frac{x^5 + 46x^4 + 9x^2 + 7}{7x^5 + 26x^3 + 8x}.$$

Using the limit laws, justify your answer.

3. Is there an  $x$  such that

$$e^x = x^4 - x?$$

Explain why or why not.

4. Let  $f$  be a continuous function from  $[0, 1]$  to  $[0, 1]$  (i.e.  $[0, 1]$  is the domain of  $f$ , and  $0 \leq f(x) \leq 1$  for each  $x$  in the domain of  $f$ ). Show that  $f$  has a *fixed point*, that is, that there exists some  $a$  with  $0 \leq a \leq 1$  such that  $f(a) = a$ . [Hint: apply the Intermediate Value Theorem to  $f(x) - x$ .]

Is the same thing true if  $f$  is any continuous function from the real numbers to the real numbers?

5. Make a function  $h$  as follows: for  $-1 \leq x \leq 1$ ,  $h(x) = x$ . For  $x$  outside of this interval, consider the sequence of numbers  $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \dots$ . Since  $|x| > 1$ ,  $|\frac{1}{x}| < 1$ , so eventually this sequence of numbers has absolute

value less than  $\frac{1}{2}$ . Let  $h(x)$  be the first term of this sequence which has absolute value strictly less than  $\frac{1}{2}$ . Using the squeeze theorem, show that

$$\lim_{x \rightarrow \pi} (x - \pi)h(x)$$

exists. Is the function  $(x - \pi)h(x)$  continuous at  $\pi$

6. Explain the following in terms of the Intermediate Value Theorem:  
At any instant, there is a point on the Earth's equator which has exactly the same temperature as the point on the opposite side of the Earth.