

Math 1A Worksheet 6

February 4th, 2008

1. Find

$$\lim_{x \rightarrow 0} \frac{\sin^2(2x)}{\sin(x) \sin(3x)}.$$

2. Calculate

$$\lim_{x \rightarrow -3^+} \frac{x^2 + 4x + 3}{|x + 3|}$$

and

$$\lim_{x \rightarrow -3^-} \frac{x^2 + 4x + 3}{|x + 3|}.$$

Let a be any real number, and let

$$f(x) = \begin{cases} \frac{x^2 + 4x + 3}{|x + 3|} & , \quad x \neq -3 \\ a & , \quad x = -3 \end{cases}$$

Explain why $f(x)$ cannot be continuous at -3 , regardless of what a is.

3. Let

$$f(x) = \begin{cases} 1 - x^2 & , \quad 0 \leq x \leq 1 \\ 1 + \frac{x}{2} & , \quad 1 < x \leq 2 \end{cases}$$

- a) Show that f is not continuous on $[0, 2]$.
b) Show that f does not take on all values between $f(0)$ and $f(2)$, that is, show that there is a number between $f(0)$ and $f(2)$ which is not a value of f on the interval $[0, 2]$.

4. Is there an x such that

$$e^x = x^4 - x?$$

Explain why or why not.

5. Make a function h as follows: for $-1 \leq x \leq 1$, $h(x) = x$. For x outside of this interval, consider the sequence of numbers $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \dots$. Since $|x| > 1$, $|\frac{1}{x}| < 1$, so eventually this sequence of numbers has absolute value less than $\frac{1}{2}$. Let $h(x)$ be the first term of this sequence which has absolute value strictly less than $\frac{1}{2}$. Using the squeeze theorem, show that

$$\lim_{x \rightarrow \pi} (x - \pi)h(x)$$

exists. Is the function $(x - \pi)h(x)$ continuous at π ?