Math 1A Worksheet 5

January 31st, 2007

1. Use the Squeeze Theorem to show that $\lim_{x\to 0} \sqrt{x^3+x^2}\sin(\frac{\pi}{x}) = 0$.

2. Calculate
$$\lim_{x\to -3^+} \frac{x^2+4x+3}{|x+3|}.$$
Explain your answer carefully.

3. Consider the function
$$f(x) = \left\{ \begin{array}{ccc} 0 & , & x \text{ irrational} \\ x & , & x \text{ rational} \end{array} \right\}.$$

a) Does
$$\lim_{x\to 0} f(x)$$
exist? If so, what is the limit? Is f continuous at 0?

b) Let a be a nonzero real number. Does
$$\lim_{x\to a} f(x)$$
exist?

c) For which real numbers a is $f(x)$ continuous at a?

4. The floor function $\lfloor x \rfloor$ is defined by: for each real number a, $\lfloor a \rfloor$ is the largest integer n such that $n \leq a$. So, for example, $\lfloor 3/2 \rfloor = 1$, since $1 \leq 3/2$ but $2 > 3/2$.

a) Let n be an integer. What is

$$\lim_{x \to n^+} \lfloor x \rfloor ?$$

What about the limit from below? Does

$$\lim_{x \to n} \lfloor x \rfloor$$

exist?

b) Now, let a be a real number that is not an integer. Does

$$\lim_{x \to a} \lfloor x \rfloor$$

exist? What is it?

c) From a) and b), for which real numbers is $\lfloor x \rfloor$ continuous?

d) Calculate

$$\lim_{x \to 1} \frac{x^2 - \frac{x}{2} - \frac{1}{2}}{x - 1}.$$

[Hint: the numerator factors!]

e) Find

$$\lim_{x \to 1} \left\lfloor \left\lfloor \frac{x^2 - \frac{x}{2} - \frac{1}{2}}{x - 1} \right\rfloor \right\rfloor .$$