Math 1A Worksheet 4

January 30th, 2007

1. Use the Squeeze Theorem to show that $\lim_{x\to 0} \sqrt{x^3 + x^2}\sin(\frac{\pi}{x}) = 0$.

2. Suppose a and δ are positive real numbers with $\delta > 0$. How do you describe all real numbers x that are within δ of a? Describe this set:
   i) graphically (on a number line),
   ii) using inequalities,
   iii) using absolute value notation, and
   iv) using interval notation.

3. Let $f(x) = x^3$. Find all the positive numbers x such that $f(x)$ is within 1 of 9. Then find a real number δ such that if x is within δ of 3, then $f(x)$ is within 1 of 9.

4. Find a $\delta > 0$ such that whenever $|x - \frac{\pi}{2}| < \delta$, we have $|sinx - 1| < \frac{1}{2}$. If you are stuck, draw a graph of $sinx$, and remember how to turn an inequality involving an absolute value into two "normal" inequalities.

5. Suppose $f(x)$ is some increasing function with $f(3) = 2$ and $f(0) = -1$. (If this is confusing, draw a graph of a function which has these properties.) Find a $\delta > 0$ so that whenever $|x - 2| \leq \delta$, we have $-1 \leq f(x) \leq 2$. For the δ you have chosen, is it true that if $|x-2| \leq \delta$, then $|f(x) - f(2)| \leq 3$?

6. Calculate
   $$\lim_{x\to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a}$$
   for any nonzero real number a.

   [Note: for those who realize this is the derivative of $\frac{1}{x}$ at a, you may *not* simply evaluate the derivative at a. This problem amounts to *showing* that the derivative of this function really is what you think it is.]

7. (Putnam 2002) Given any five distinct points on the surface of a sphere, show that we can find a closed hemisphere which contains at least four of them. (Note: this has *absolutely nothing* to do with our class.)