Math 1A Worksheet 3

August 31st, 2007

1. a) Give an example of two functions $f(x)$ and $g(x)$ such that $\lim_{x\to 0} f(x)+g(x)$ exists even though neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

   b) Give an example of two functions $f(x)$ and $g(x)$ such that $\lim_{x\to 0} f(x)g(x)$ exists even though neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

2. Graph the following functions of x. Do NOT do so by plotting points.
   i) $\cos(\frac{x}{2\pi})$
   ii) e^{x+1}
   iii) $\sin(\frac{1}{x})$, where $0 < x$.

3. Consider the function $f(x) = \sin(\frac{1}{x})$ which we graphed in problem 1. Let ϵ be any positive (non-zero!) real number. What values can $f(x)$ have for $0 < x < \epsilon$? Does
$$\lim_{x\to 0^+} \sin\left(\frac{1}{x}\right)$$
exist?

4. Suppose
$$\lim_{x\to a^+} f(x) \quad \text{and} \quad \lim_{x\to a^-} f(x)$$
exist. Does $\lim_{x\to a} f(x)$ have to also exist? If so, explain why; if not, give a counterexample.

5. For what kind of functions can one evaluate the limit $\lim_{x\to a} f(x)$ just by plugging in $x = a$? Give an example of a function $f(x)$ for which the limit $\lim_{x\to a} f(x)$ can be evaluated, yet for which this substitution doesn't work.

6. Use the Squeeze Theorem to show that $\lim_{x\to 0} \sqrt{x^3 + x^2}\sin(\frac{\pi}{x}) = 0$.

7. List all functions f which are both even and odd.