Math 1A Worksheet 29

November 21th, 2007

1. Let's calculate $\int_1^2 \ln x \, dx$. We can't do this directly, since we don't know how to find an antiderivative. Instead, do the following:
   a) Draw a graph showing $\int_1^2 \ln x \, dx$ as an area.
   b) Reflect this picture across the line $y = x$. What function does $\ln x$ correspond to when reflected in this way? Label as much as you can on the new, reflected picture.
   c) Use this new picture to calculate the area.

2. Let
$$f(x) = \begin{cases} \frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0. \end{cases}$$
   a) Find $\int_1^2 f(x) \, dx$.
   b) Let t be a small real number, greater than 0. Draw a picture that shows the relationship between $\int_0^1 f(x) \, dx$ and $\int_t^1 f(x) \, dx$, and use this picture to write an inequality relating these two integrals. Find $\lim_{t \to 0^+} \int_t^1 f(x) \, dx$. What does this say about $\int_0^1 f(x) \, dx$?
   c) Show the same thing you showed in part b), this time using Riemann sums. [Hint: think about x_1^*.]

3. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t) \, dt$ represent?

4. a) Explain in words the difference between "displacement" and "distance traveled."
   b) Suppose a particle moves with velocity $v(t)$, which varies continuously over time. Use the fundamental theorem of calculus to write both the displacement and distance traveled by the particle as functions.
   c) Write $s(t)$ for the position of the particle at time t. We decided before that the average velocity between times a and b is $\frac{s(b)-s(a)}{b-a}$.

Using the FTC, show that this agrees with the answer you get if you use the "integral definition" of the average.