

Math 1A Worksheet 28

April 24th, 2007

1. The curves $y = x$ and $y = x^2$ meet in two points and form the boundary of some shape. Find the area of this shape.
2. (Repeat from last time) Recall that last time, we showed that

$$\sum_{i=0}^{n-1} ar^i = \frac{a(r^n - 1)}{r - 1}.$$

Let's use this to calculate $\int_0^1 e^x$ using Riemann sums. (We know from the fundamental theorem of calculus that we expect to get $e - 1$, but this is neat to do another way.)

- a) Partition $[0, 1]$ into n equal intervals of length $\frac{1}{n}$. Using the *left-hand* endpoint of each interval, write a Riemann sum approximating the above integral.
 - b) Use the above summation formula to rewrite the Riemann sum as a simpler expression.
 - c) Now take the limit as $n \rightarrow \infty$. You'll need to use l'Hospital's rule.
3. Let's calculate $\int_1^2 \ln x \, dx$. We can't do this directly, since we don't know how to find an antiderivative. Instead, do the following:
 - a) Draw a graph showing $\int_1^2 \ln x \, dx$ as an area.
 - b) Reflect this picture across the line $y = x$. What function does $\ln x$ correspond to when reflected in this way? Label as much as you can on the new, reflected picture.
 - c) Use this new picture to calculate the area.