Math 1A Worksheet 24

April 11th, 2008

1. Story problem time! Sophie Snell, a lifeguard, is sitting at her lifeguard post, 20 meters from the water's edge. She spots a swimmer in trouble; he's floundering 20 meters out into the water, and the point on the beach nearest to him is 40 meters away from the point in the water nearest to Sophie.
   a) Draw a picture to illustrate the above, or ask me very nicely to draw a picture.
   b) On land, Sophie can run at 6 meters per second, while in the water, she moves at only 2 meters per second. Given this, discuss where on your picture Sophie should enter the water to reach the swimmer fastest. No equations yet!
   c) Now, let x be the distance between the point on the water closest to Sophie's lifeguard post and the point where Sophie enters the water. Add x to your picture. Set up an equation for the time it takes Sophie to reach the swimmer in terms of x. Don't solve this yet!
   d) Take a derivative of the equation you've found and set it equal to 0. Note that there will only be one solution. Now, use the equation you get to find a relationship between the cosines of the angles between the shoreline and each of the two legs of Sophie's path. Use this to make a rough sketch of the shortest path.
   e) Now solve for x to an accuracy of 1 meter (by method of your choice; calculator allowed).

2. You are going to make a box with a square base and no top. If the surface area of the resulting box is going to be 100 square inches, what is the largest volume of box you can make? [Note: the answer isn't pretty.]

3. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

4. Find all functions $f(x)$ with $f''(x) = e^x$.

5. (Berkeley Prelim Spring 2001) Suppose $f(x)$ is a continuous function, defined for all real numbers, and suppose moreover that f is periodic with period 1, i.e. $f(x+1) = f(x)$ for all x. Show that $f(x)$ is bounded from above and below (i.e. there exist constants m and M such that $m \leq f(x) \leq M$ for all x), and show that f has an absolute maximum and minimum.