Math 1A Worksheet 23

October 31st, 2007

1. Consider the quadratic polynomial $f(x) = ax^2 + bx + c$ where a, b, and c are some real numbers and $a \neq 0$. Show that f has exactly one critical point and no inflection points. When is f concave up? When is it concave down?

2. Find
$$\lim_{x \to 0} \frac{(x^2)^{x^2} - 1}{x}.$$

3. Let $f(x)$ be a differentiable function defined for all real numbers. Recall that a *fixed point* of f is a number a such that $f(a) = a$. Suppose that for every x, we have $f'(x) \neq 1$. Use the MVT to show that f has at most one fixed point. [Hint: use an auxillary function whose zeros correspond to fixed points of f. We did something similar to this before.]

4. Let $f(x)$ be a quadratic polynomial as in Problem 1. Suppose f has two real zeros r and s. Show that $f'(r) + f'(s) = 0$. Show also that the unique critical point of f occurs halfway between r and s.